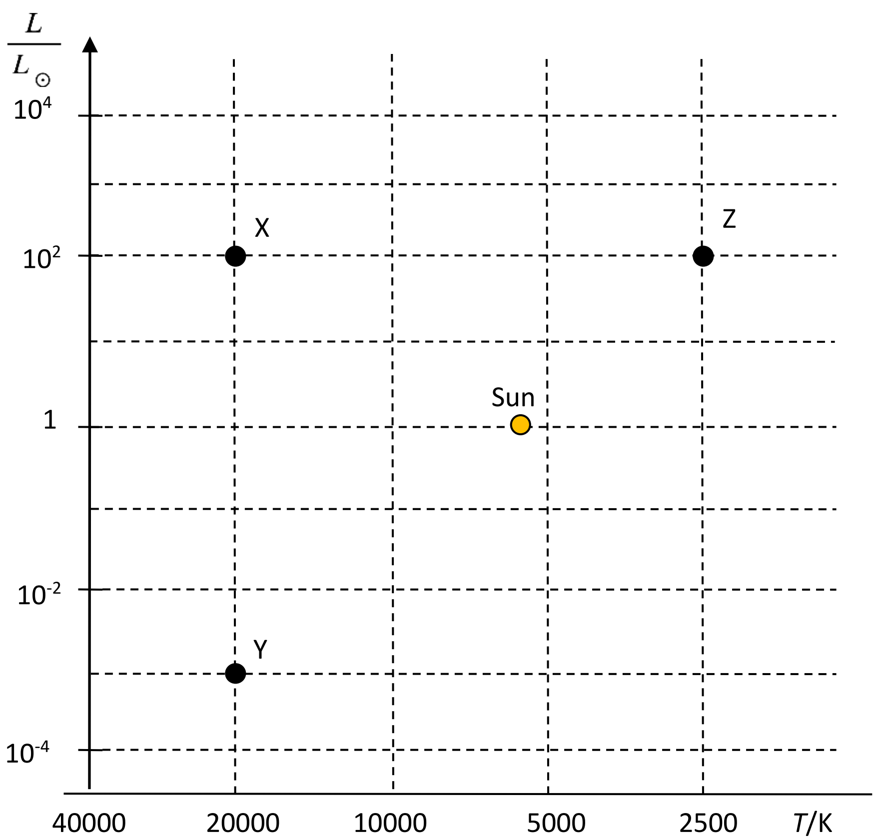
Teacher notes

Topic E

The HR diagram



(a) Identify (i) the main sequence, (ii) the region of red giants, (iii) the region of white dwarfs and (iv) the instability region.

(b) X and Y have the same luminosity even though X has a much larger temperature. Explain this observation.

(c) What is the ratio of radii $R_Z : R_X : R_Y$ for Z and X and Y?

(d) Describe and draw the evolutionary path of the Sun and of star X.

(e) Describe how star X and star Y maintain equilibrium.

Answers

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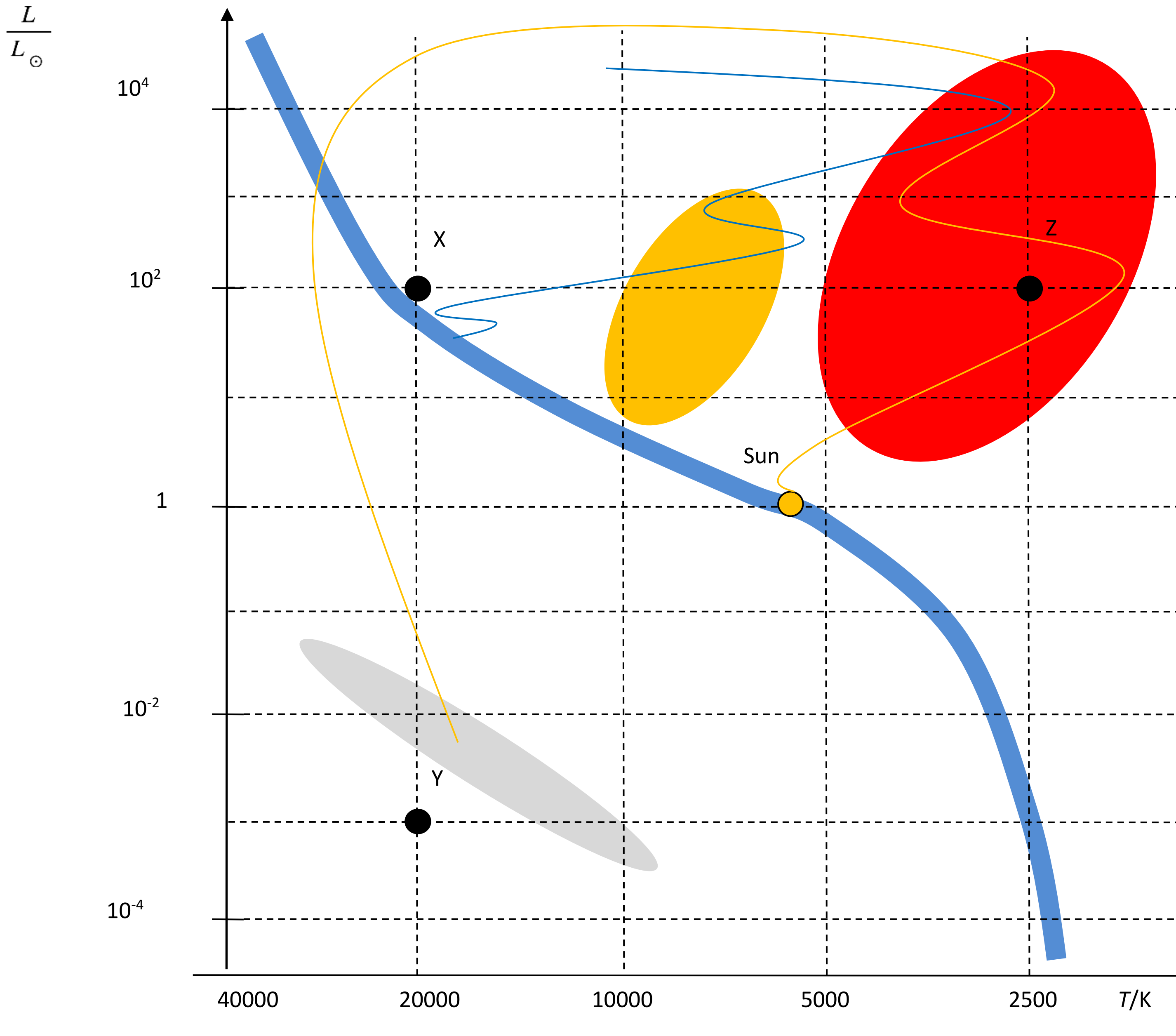
(a)



$\frac{L}{L_\odot}$
10^4
10^2
1
10^{-2}
10^{-4}
X
Y
Z
Sun
40000
20000
10000
5000
2500
T/K

(b) Z must have a much larger surface area.

(c) $\frac{\sigma 4\pi R_Z^2 \times 2500^4}{\sigma 4\pi R_X^2 \times 20000^4} = 1 \Rightarrow \frac{R_Z}{R_X} = (\frac{20000}{2500})^2 = 64$; $\frac{\sigma 4\pi R_X^2 \times 20000^4}{\sigma 4\pi R_Y^2 \times 20000^4} = 10^5 \Rightarrow \frac{R_X}{R_Y} = \sqrt{10^5} \approx 320$. Hence

$R_Z : R_x : R_Y \approx 20500 : 320 : 1$

(d) See diagram.

(e) X: pressure (gas pressure and radiation pressure) created by the energy produced in nuclear fusion

Y: electron degeneracy pressure.

Why are lines of constant stellar radius straight lines on the HR diagram?

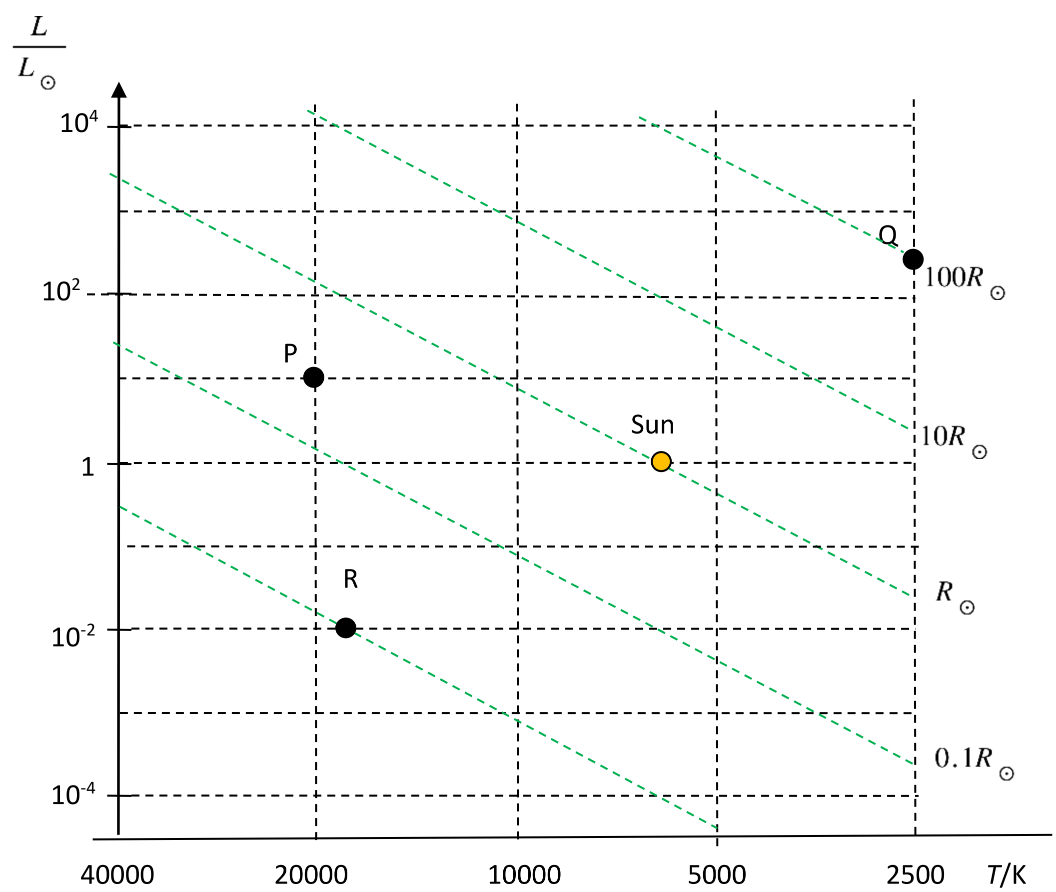


Diagram 1

(Stars P, Q and R will be dealt with shortly.)

We must understand that the HR diagram is a plot of log *L* versus log *T*. So, the diagram above is actually:

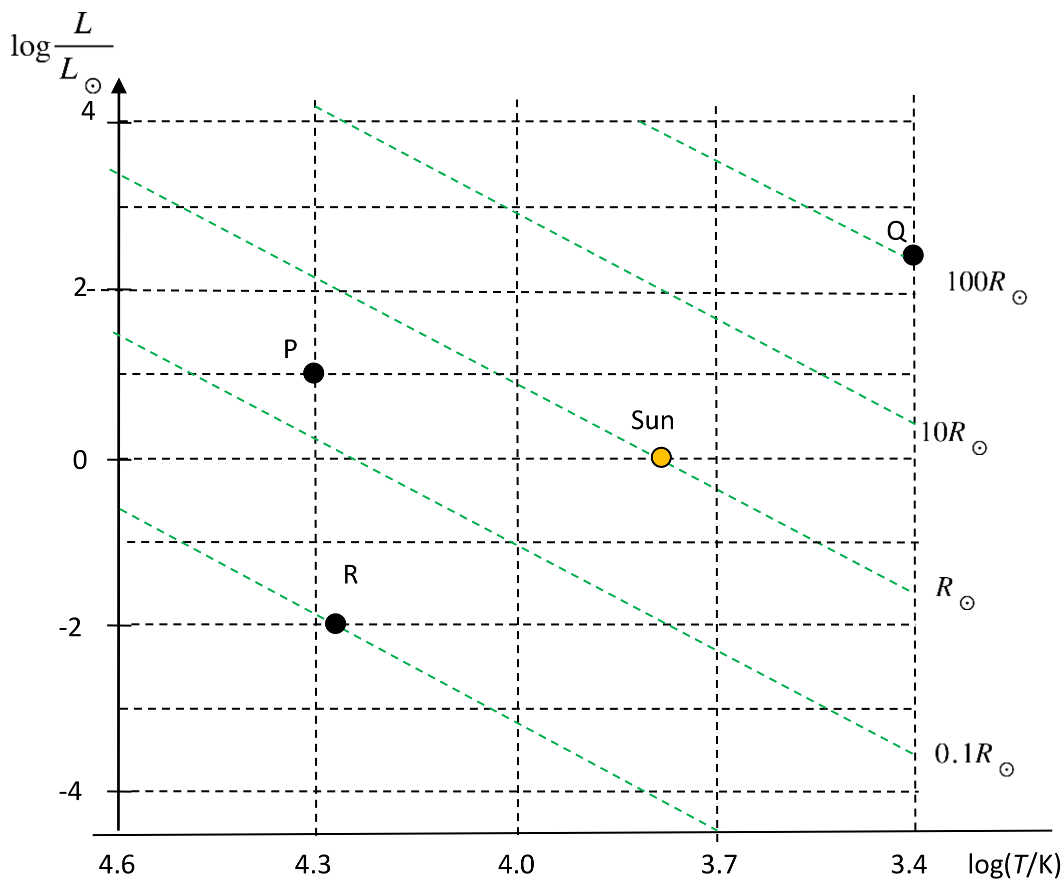


Diagram 2

In other words, **the numbers on the axes of Diagram 2 are the logs of the numbers in Diagram 1**. Conversely, **the numbers on the axes of Diagram 1 are the numbers on the axes of Diagram 2 raised to the power of 10**.

For example, $\log 5000 = 3.7$. Precisely because the scales are logarithmic you cannot easily find the temperature of a star that does not fall on the grid. For example, consider a star whose coordinate is half-way between 5000 K and 10000 K in Diagram 1. Its temperature is **not** 7500 K. To find it go to diagram 2. Half-way between $\log 5000 = 3.7$ and $\log 10000 = 4.0$ is 3.85 so the temperature is $T = 10^{3.85} = 7080\text{ K}$. Similarly, let us say we wanted to find the temperature of the Sun. From the second diagram the log T coordinate of the Sun is about 3.75. This means that $\log T = 3.75$ and so $T = 10^{3.75} = 5600\text{ K}$.

(Because we exponentiate, uncertainties tend to grow and we have inaccurate results. The actual coordinate is 3.762 and $T = 10^{3.762} \approx 5780\text{ K}$. You see how a small difference in the coordinate results in very different answers.)

Looking at the constant radius line through the Sun we find a gradient of:

$$\frac{3.3-(-1.5)}{4.6-3.4}=4$$

This is expected because since $L=\sigma 4\pi R^2 T^4$ it follows that

$\log\frac{L}{L_\odot}=\log\frac{\sigma 4\pi R^2 T^4}{\sigma 4\pi R_\odot^2 T_\odot^4}=2\log\frac{R}{R_\odot}+4\log\frac{T}{T_\odot}$. For R = constant,

$$\log\frac{L}{L_\odot}=c+4\log\frac{T}{T_\odot}$$

where $c=2\log\frac{R}{R_\odot}$ is a constant since R is. In a graph of $\log\frac{L}{L_\odot}$ against $\log\frac{T}{T_\odot}$ this would be a straight line with positive slope of 4 if T was increasing to the right as in a normal graph. But T is increasing to the left so this makes the straight line look like it has a negative slope.

For the line through the Sun $c=2\log\frac{R_\odot}{R_\odot}=2\log 1=0$.

We can now ask for (using the standard Diagram 1): the radius of P, the luminosity of Q and the temperature of R. (This is most likely not something that could be asked on an IB exam but it could be useful to someone doing an IA or EE.)

In the following we use $\log\frac{L}{L_\odot}=2\log\frac{R}{R_\odot}+4\log\frac{T}{T_\odot}$ with $T_\odot=5780\text{ K}$.

For P:

$\log 10=2\log\frac{R}{R_\odot}+4\log\frac{20000}{5780} \Rightarrow \log\frac{R}{R_\odot}=-0.578$. Hence $\frac{R}{R_\odot}=10^{-0.578}=0.26$.

For Q:

$\log\frac{L}{L_\odot}=2\log 100+4\log\frac{2500}{5780}=2.544$. Hence $\frac{L}{L_\odot}=10^{2.544}=3.5\times 10^2$.

For R:

$\log 10^{-2}=2\log 10^{-2}+4\log\frac{T}{5780} \Rightarrow \log\frac{T}{5780}=0.5$. Hence $\frac{T}{5780}=10^{0.5} \Rightarrow T=1.8\times 10^4\text{ K}$.

These results are consistent with Diagram 2.